Streamline generation code for particle dynamics description in numerical models of turbulence

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ABSTRACT
Streamline Version 4 is a versatile Fortran 77 & C++ program for calculating charged test particle trajectories or field-lines for user-specified fields using the test-particle method. The user has the freedom to specify any type of field (analytical, tabulated in files, time dependent, etc.) and maintains complete control over initial conditions of trajectories/field-lines and boundary conditions of specified fields. The structure of Streamline was redesigned from previous versions in order to know not only particle or field-lines positions and velocities at each step of the simulations, but also the instantaneous field values as seen by particles. This was made to compute the instantaneous value of the particle’s magnetic moment, but other applications are possible too. Accuracy tests of the code are shown for different cases, i.e., particles moving in constant magnetic field, magnetic plus constant electric field and wavefield. In addition in the last part of the paper we concentrate our discussion on the study of velocity space diffusion of charged particles in turbulent slab fields, paying attention to the discretization of the fields and the temporal discretization of the dynamical equations. The diffusion of charged particles is a very common topic in plasma physics and astrophysics since it plays an important role in many different phenomena such as stochastic particle acceleration, diffusive shock acceleration, solar energetic particle propagation, and the scattering required for the solar modulation of galactic cosmic rays.

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1. Introduction

Plasma consists of positive charged ions of mass $m_i$ and negative charged electrons of mass $m_e$. Since $m_i \gg m_e$, there are many intrinsic spatial and time scales in a plasma system, even if a uniform background environment is considered. For time scales smaller than the intrinsic time scales, it is hard for the system to reach a thermal dynamic equilibrium state and kinetic effects might become important. This also means that the typical length scale of the system, $L$, is comparable to the characteristic length scale of a given species (ions and/or electrons), i.e., the Larmor radius, $r_L = v_{th} / \Omega$, in which $v_{th}$ is the particle’s thermal velocity and $\Omega = qB/mc$ is the particle’s gyrofrequency.

Generally, two different numerical approaches may be considered in order to study plasma evolution: the kinetic approach and the fluid approach. Kinetic simulations are useful to study the non-linear evolutions of wave–particle interactions in the phase–space on spatial and time scales on which kinetic effects are not negligible. On the other hand, a fluid simulation code can provide reasonable and relatively quick results when the kinetic effects become unimportant. Depending on their resolutions in phase–space, we can classify the most common plasma simulations as follows:

- Fluid Simulations
  - MHD code: $L \geq 10^3 r_L$
  - Two-Fluid code: $10^3 r_L \geq L \geq 10 r_L$

- Kinetic Simulations
  - Hybrid code (fluid electrons & kinetic ions): $10 r_L \geq L \geq r_{le}$
  - Full particle code: $r_L \geq L \geq r_{le}$
  - Test particle code: when a strong magnetic field is present
  - Vlasov code: $r_L \geq L \geq r_{le}$

where $L$ is the typical length scale of the system and $r_{le}$ are the typical ion/electron gyroradii. When $L$ approaches the ion thermal gyroradius, ions become demagnetized and the plasma can no longer behave as a simple fluid. As a consequence, the usual
MHD description breaks down in favor of a more complex, kinetic, plasma description.

In this paper we concentrate our discussion on the test-particle methods, so the electromagnetic fields are treated as prescribed and particles can be treated fully independently from each other in the simulation. In the absence of interactions and feedback from calculated particle trajectories, the results are generally not self-consistent. Test-particle calculations can be used to study a broad class of problems in space physics and astronomy, such as particle transport, energization and dynamics in complex systems, for which a fully consistent kinetic calculation is not practical. Indeed, test-particle simulations represent a complementary approach between fluid and fully kinetic models. Fluid models are powerful tools for modeling complex systems, while accounting for realistic geometry together with multiple physical processes. However, they are limited to the description of macroscopic properties of plasmas in terms of the local distribution functions momenta. On the other hand, kinetic models provide detailed information on particle dynamics but, because of their complexity, they are limited to relatively simple geometries in the range of physical processes that they can account for. In that context, test-particles provide a useful bridge between the two approaches. Using approximate fields obtained from macroscopic models or analytical field expressions, they can be applied to assess kinetic effects in complex systems under realistic conditions.

The version of the Streamline code described here, is part of a series of programs developed at the University of Delaware within Dr. Matthaeus’s group in the Department of Physics and Astronomy and Bartol Research Institute. The replacement of a tested code is generally frowned upon, but the main ordinary differential equations (ODE) integration in all versions relied on modified code from Numerical Recipes. Thus, the ODE integration is performed using published routines that are operationally unchanged from their documented source code. Streamline has the ability to calculate static field-lines, particle trajectories or, more generally, to solve the first order ODE:

\[
\frac{dx}{ds} = f(x),
\]

where \( x = (x_1, x_2, \ldots, x_N) \) is an \( N \)-dimensional generalized position, \( s \) is a generalized arc length parameterization, and \( f(x) = (f_1(x), f_2(x), \ldots, f_N(x)) \) is a vector field determinable at every generalized position, \( x \). As an illustration, consider a particle with mass \( m \) moving under the time-dependent force, \( F(x, t) \). In this case, \( s \equiv t \), and we have:

\[
x = (x, y, z, v_x, v_y, v_z),
\]

\[
f = (v_x, v_y, v_z, a_x, a_y, a_z)
\]

where \( a = F/m \) and \( N \equiv 6 \). This system is usually written as two coupled ODE's:

\[
\frac{dx}{dt} = v(x, t),
\]

\[
\frac{dv}{dt} = a(x, t).
\]

This code is a MPI parallel implementation of a versatile algorithm for computation of streamlines, magnetic field lines or charged particle trajectories. Each of these has in common that a set of generalized “trajectories” is generated, beginning from a set of specified initial conditions. The trajectories are generated by integrating a set of ODE’s, either three (for streamlines and field lines) or six (for charged particle orbits). In each case one or more vector fields, appearing as coefficients in the ODE’s, must be specified. Physically, what is required for each different case is:

- Streamlines: specify a velocity field and an initial position;
- Magnetic Field Lines: specify a magnetic field and an initial position;
- Charged Particle Orbits: specify a magnetic field, and electric field, and an initial position and velocity.

The ODE’s are integrated using an adaptive step fourth order Runge-Kutta method, with a fifth order error estimate [1]. The entire simulation run is broken down into a number of substeps, each with time interval \( \delta t \ll T_{max} \) where \( T_{max} \) is the total length of the run. The routine steps each particle in turn through the time interval, \( dt \), while maintaining a local relative accuracy of \( racc = 10^{-9} \) at each step. This process is repeated until the run ends and all particles are stepped through the time interval for the entire run, \( T_{max} \). Several standard cases are built in at present, i.e., field line or charged particle orbit equations and the magnetic field input model (slab, slab plus 2D, data read from file). Several test cases are also installed in the code. Provisions are made for user-supplied ODE’s (routine DERIVS) and user-supplied electromagnetic fields (routine EMFIELDS), that are externallylinked and communicate with the internal routines through standard data structures. Streamline Version 4 is capable of handling analytic fields, tabulated fields from external files, or no fields at all, so long as Eq. (1) is fully defined. These fields are expected to be electromagnetic but this is not assumed.

This piece of software is a very versatile field generation code, with many features that give advanced users all the power through the various input files. This also has the possibility for extending and/or adding new programs and algorithms. The whole MPI part could be used as a wrapper around different versions of electromagnetic fields and various derivatives. This is done by considering the whole setup as a black-box and writing customized “emfids” and “derivs” subroutines by following the interfaces provided in this program. The code works programming the master node to pass out “jobs” to the worker nodes. Each job includes the initial data and some other main parameters. Load balancing is achieved in a standard way: when a node completes a job, it asks the master if there is another job to do. Results of each job are written to disk.

The paper is organized as follows: in Section 2 we restrict the application of Streamline Version 4 to particles. In Section 3 we perform some initial tests on the accuracy of the code. In Sections 4 and 5 we give a short overview of the physical problem concerning wave–particle interaction and then we present numerical results from particles moving in a two circularly polarized waves field. Section 6 summarizes some features of a particularly simple model of plasma turbulence, the so-called slab model and Section 7 shortly describes the velocity space diffusion by particles interacting with weak plasma turbulence. In Section 8, we present the numerical results from several runs in order to study periodicity effects, the spatial grid resolution and energy conservation. Finally, we give the conclusions.

2. Application to particles

The behavior of a test-particle is described by its time-dependent position, \( \mathbf{r}(t) \), and three dimensional velocity, \( \mathbf{v}(t) \), that are advanced according to \( d\mathbf{r}/dt = \mathbf{v} \) and the Lorentz force equation:

\[
m \frac{d\mathbf{v}}{dt} = q \left[ \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right].
\]

The essence of test-particle simulations is that the electric, \( \mathbf{E} \), and the magnetic, \( \mathbf{B} \), are not influenced by the particle motion.

The dimensionless units [2] include an arbitrary length scale, \( \lambda \), a characteristic Alvén speed, \( v_A \), and a unit transit time,
The parameter \( \beta = \Omega \tau_A \) (\( \alpha \) parameter in [3]), which arises in Eq. (7) as a consequence of our choice of normalization, couples particle and field relative spatial as well as temporal scales and provides a particularly useful means to relate our abstract numerical experiment to real space and astrophysical plasma situations. For example:

\[
\beta = \Omega \tau_A = 2\pi \frac{\tau_A}{\tau_g} = \frac{\omega_p \lambda}{c} = \omega_p \tau_c
\]

where \( \tau_g = 2\pi / \Omega \) is the test particle gyroperiod; \( \omega_p = (4\pi n_0 q_i^2/m_i)^{1/2} \) is the ion plasma frequency in the background plasma, where \( n_0 \) and \( m_i \) are, respectively, the charge and mass of a background ion; \( \tau_c = \lambda / c \), where \( c \) is the speed of light. Other important test-particle parameters can be expressed in terms of \( \beta \) in a simple way. For example, the test particle gyroradius \( r_g \) and maximal gyroradius \( r_t \) are, respectively, given by Mace et al. [4]

\[
\rho_g = \frac{v_u}{\Omega} = \frac{v_u / v_A}{\beta}, \quad r_t = \frac{v}{\Omega} = \frac{v / v_A}{\beta}.
\]

In general \( \beta \gg 1 \), that is the turbulent time scales are much slower than the typical particle gyroradius [5].

3. Test of accuracy of Streamline code

In order to substantiate the accuracy of numerical results coming from more complex electromagnetic field configurations, we test the Streamline code for different and simpler cases:

1. Particles moving in a constant magnetic field, \( \mathbf{B} = B_0 \mathbf{e}_z \)
2. Particles moving in a constant magnetic field, \( \mathbf{B} = B_0 \mathbf{e}_z \), plus a constant electric field, \( \mathbf{E} = E_0 \mathbf{e}_x \)
3. Particles moving in a circularly polarized wave field.

For the first two cases, the exact solution of the problem is known. Thus, it is possible to compare directly the analytical solution with the numerical results. For the last case, because no electric fields are present in the system, the condition that guarantees the accuracy of our simulations is the energy conservation.

### Table 1

<table>
<thead>
<tr>
<th>Normalization quantities</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arbitrary length scale</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>Characteristic Alfvén speed</td>
<td>( v_A )</td>
</tr>
<tr>
<td>Unit transit time</td>
<td>( \tau_A = \lambda / v_A )</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>( B_0 = \sqrt{4\pi \rho} )</td>
</tr>
<tr>
<td>Electric field</td>
<td>( E_0 = (\tau_A/c) B_0 = v_A^2 \sqrt{4\pi \rho} / c )</td>
</tr>
</tbody>
</table>

3.1. Particles moving in a constant magnetic field, \( \mathbf{B} = B_0 \mathbf{e}_z \)

As we know from the classical particle motion theory, a particle with charge \( q \) moving with a velocity \( \mathbf{v} = v_1 + v_\perp \) in a uniform magnetic field \( B_0 \), experiences the so-called Lorentz force \( \mathbf{F}_L \) perpendicular to both the particle velocity \( \mathbf{v} \) and the magnetic field \( \mathbf{B}_0 \), so that it does no work on the particle. The analytic solution of this problem is given by:

\[
\begin{align*}
\mathbf{v}_x(t) &= v_{0x} \cos(\Omega t) + v_{0y} \sin(\Omega t) \\
\mathbf{v}_y(t) &= v_{0y} \cos(\Omega t) - v_{0x} \sin(\Omega t) \\
\mathbf{v}_z(t) &= v_{0z} = \text{const}
\end{align*}
\]

where \( \Omega = qB/m \) and \( v_{0x}, v_{0y} \) and \( v_{0z} \) are the initial velocities in \( x, y \) and \( z \), respectively. Thus the charged particle moves on a circle in the \( x - y \) plane around the magnetic field \( B_0 \), while the parallel velocity component \( v_z \) carries the particle along the magnetic field lines creating a helical trajectory.

3.2. Particles moving in a constant magnetic field, \( \mathbf{B} = B_0 \mathbf{e}_z \), plus a constant electric field, \( \mathbf{E} = E_0 \mathbf{e}_x \)

The analytic solution for this case can be written as:

\[
\begin{align*}
\mathbf{v}_x(t) &= v_{0x} \cos(\Omega t) + \left(v_{0y} + \frac{E_0}{B_0} \right) \sin(\Omega t) \\
\mathbf{v}_y(t) &= \left(v_{0y} + \frac{E_0}{B_0} \right) \cos(\Omega t) - v_{0x} \sin(\Omega t) - \frac{E_0}{B_0} \\
\mathbf{v}_z(t) &= v_{0z} = \text{const}
\end{align*}
\]

Since an electric field is present in the \( x \)-direction, particles drift in the \( y \)-direction and their trajectories assume a cycloidal shape.

In contrast to the previous case, the energy is oscillating and not constant.

We define for both cases the relative error as:

Relative Error \( = \frac{\left| v_{num}^2 - v_{sol}^2 \right|}{v_{sol}^2} \)

where \( v_{num} \) and \( v_{sol} \) are the magnitude of velocity from the analytical solution and from numerical results, respectively. We test 10 particles injected with random initial points and initial velocities given by \( v_{0x} = 10 v_{num}, v_{0y} = 0 \) and \( v_{0z} = 0 \); thus \( v_{num} \) is the average velocity for 10 particles.

For the two cases described above, the results are collected at various locale relative accuracy values \( racc \). When particles move in a uniform magnetic field, the relative error increases linearly with time in a log–log scale with a slope \( = 1 \), as shown in Fig. 1(a). When particles move in uniform magnetic and electric field, Fig. 1(b), the error grows with the same slope of the previous case. However for \( racc = 10^{-10} \) round-off effects appear.

The relative error depends on the parameter \( racc \) and increases linearly with time at the beginning of the simulation. For the two tested cases at \( 10^5 \) gyroperiods, which is a reasonably long time to study many physical processes occurring in our systems, we get a relative error of order \( 10^{-4} \) for \( racc = 10^{-5} \). Thus \( racc = 10^{-3} \) can be chosen as the best value for the accuracy parameter in our simulations.

Once \( racc = 10^{-9} \) is fixed, additional tests are done in order to verify if the relative error remains of the same order also when some typical parameters, such as particle’s velocity \( v \) and \( \beta \) are varied. In Fig. 2 the relative error of particle trajectories in a uniform magnetic field is shown for different values of particle velocities, \( v = (1 - 10 - 1000) v_A \) (top row) and for different values of the \( \beta \) parameter, \( \beta = (1 - 10 - 10^5) \) (bottom row). After \( 10^5 \) particle gyroperiods the relative error is always of the order of \( 10^{-4} \) for all the cases.
3.3. Particles moving in a circularly polarized wave field

In this section we consider the ion’s motion in the presence of a constant magnetic field, \( B_0 = B_0 \mathbf{e}_z \), plus a perpendicular circularly polarized wave. We use the left-hand polarized component of the wave field rotating in the same sense as the ions. The handness is important to properly study the resonance wave–particle interaction. Indeed, left-hand positive ions interact with left-handed waves, while right-hand negative electrons interact with right-handed waves. The resulting magnetic field is given by:

\[
B = \delta B_x \cos(k_0 z) - \delta B_y \sin(k_0 z) + B_0 \mathbf{e}_z,
\]

where the mean magnetic field \( B_0 \) is chosen in the \( z \)-direction, \( \delta B_x \) and \( \delta B_y \) are the amplitudes of the wave in the \( x \)- and \( y \)-directions, and \( k_0 \) is the wavevector assumed to lie only in the \( z \)-direction. We also assume \( \delta B_x = \delta B_y = \delta B \) for the r.m.s. average values.

In this case a different type of analysis is used to test the accuracy of the numerical results. That is, because a static magnetic field does no work on a charged particle, the energy will be conserved (this is also case 1 described above). Therefore, it is possible to compare the magnitude of velocity \( v \) at each simulation time with its initial value \( v_{in} \). The relative error is defined as:

\[
\text{Relative Error} = \frac{v^2 - v_{in}^2}{v_{in}^2},
\]

where \( v_{in} \) and \( v \) are averaged over 10 particles.

The results are shown in Fig. 3 for different values of wave amplitude, \( \delta B/B_0 = 0.001 \) (Fig. 3(a)) and \( \delta B/B_0 = 0.1 \) (Fig. 3(b)), two values of particles velocity, \( v = (1 - 10^{-2})v_A \), and for \( \beta = (1 - 10^4) \). When the \( \beta \) value is unchanged and equal to \( 10^4 \) (top row), tests are performed varying the velocity value: \( v = 1v_A \) is shown with the black line and \( v = 10v_A \) with the red line. On the other hand, when we fixed \( v = 10^3 \) (bottom row), two different \( \beta \) values are used in the test simulation, which are \( \beta = 1 \), black line, and \( \beta = 10^4 \), red line. Although for these cases the simulations last for \( 10^3 \) particle gyroperiods, the relative error for all the considered cases is of the order of \( 2 \times 10^{-6} \), that corresponds to the relative error obtained in the previous analysis after only \( 10^4 \) gyroperiods. These additional tests confirm that our results are valid with an accuracy of \( 10^{-9} \).

4. Short overview of wave–particle interaction

In space plasma the collision time between charged particles is generally very long in comparison with the characteristic time scale of the system, namely, the inverse of the plasma frequency.
or cyclotron frequencies, and therefore the plasma can be treated as collisionless. This would imply that there is virtually no dissipation, as particle–particle collisions are infrequent. However, the presence of waves in collisionless plasma can introduce finite dissipation in the system.

It can be shown rather easily that, if the amplitude of the magnetic fluctuations at a given time scale is clearly smaller than the mean magnetic field (averaged over the time scale of the fluctuation), a perturbation approach called quasilinear approximation [6–8] is applicable to study the particle motion in a perturbed electromagnetic field. The guiding center reaction to the fluctuations turns out to be a resonant one, so that only fluctuations fulfilling certain resonance conditions contribute to the particle motion. The condition for wave–particle resonance is given by:

$$\omega - k_i v_i = n\Omega$$

(13)

where $\omega$ is the wave frequency, $k_i$ and $v_i$ are respectively the wavevector and the particle velocity along the mean magnetic field $B_0$, and $\Omega = qB_0/mc$ is the particle gyrofrequency. The $n = 0$ is the so-called Landau resonance; the $n = \pm 1, \pm 2, \ldots$ are the cyclotron resonances. If $n = 0$, resonance occurs when $\omega = k_i v_i$, so particles “surf” along the wave. In 1946 Landau [9] showed that plasma waves in unmagnetized collisionless plasma suffer a damping due to wave–particle interactions, just called Landau damping. Instead, if a particle is moving in a perpendicular wave field in presence of a strong magnetic field, $B_0 = B_0 e_z$, it will interact strongly with the wave when its streaming velocity is such that it is affected by the Doppler-shifted wave at its cyclotron frequency or its harmonics under the assumed steady conditions.

It is easy to demonstrate that the particle’s response to the perturbation is always periodic, except when the Doppler-shifted frequency in the frame moving with the particle’s parallel velocity is exactly equal to the cyclotron frequency. In this case, the perpendicular electric force due to a wave remains in phase with the rotating particle cyclotron motion and the particle’s response is secular and, over short times, non-oscillatory.

Charged particles are scattered by the wave fields, changing their pitch angles and energies through this process. The pitch angle, $\theta$, of a particle is defined as the angle between the direction of the magnetic field line and the particle’s spiral trajectory:

$$\tan \theta = \frac{v_{\perp}}{v_{\parallel}}.$$

(14)

Scattering from magnetic fluctuations tends to cause the evolution of particle’s pitch angle distribution towards isotropy.

The resonant condition can also be expressed in terms of the $\beta$ parameter defined in Section 2. For the static case, the subject of our study, it can be written as:

$$k_{res} \lambda = \frac{n\beta}{\mu(v/v_A)} = \frac{n\beta}{(v_i/v_n)\lambda},$$

(15)

where $\mu = \cos \theta = v_{\parallel}/|v_{\parallel}|$ is the cosine of pitch angle.

4.1. Expected behavior at resonance

Let us consider the ion dynamic in the interaction with a left-hand circularly polarized wave and assume that a constant background magnetic field is imposed in the $z$ direction, $B_0 = B_0 e_z$.

In a stationary case the magnetic field is:

$$B = \delta B_0 \cos(k_0 z) - \delta B_1 \sin(k_0 z) + B_0 e_z,$$

(16)

where $\delta B_0$ and $\delta B_1$ are the wave amplitudes in the $x$- and $y$-directions and $k_0$ is the wavevector assumed to lie only in the $z$-direction. We also assume $\delta B_0 = \delta B_1 = \delta B$ for the r.m.s. average values.

In the quasilinear approximation, particle motion is well described by the unperturbed motion, i.e., the classical helical trajectory in a constant magnetic field. Thus particle velocity is given by:

$$v = (v_{\perp} \sin(\Omega t) e_x, v_{\perp} \cos(\Omega t) e_y, v_i e_z).$$

(17)

If resonance between particle and wave occur, we will observe that $v_x(v_i)$ is in phase with $-B_y(B_z)$.

5. Particles moving in a two circularly polarized waves field

In this section the resonant interaction between ions and two perpendicular circularly polarized waves is analyzed in detail. For this reason, just the left-hand polarized component of the waves is considered. The fields are assumed to be static and a constant background magnetic field $B_0 e_z$ is also present. The resulting magnetic field can be written as:

$$B = 2\delta B_0 \cos \left[ \frac{(k_1 + k_2)z}{2} \right] \cos \left[ \frac{(k_1 - k_2)z}{2} \right] e_x - 2\delta B_1 \sin \left[ \frac{(k_1 + k_2)z}{2} \right] \cos \left[ \frac{(k_1 - k_3)z}{2} \right] e_y + B_0 e_z,$$

(18)

where $\delta B_0$ and $\delta B_1 (i = 1, 2)$ are wave amplitudes in $x$ and $y$ directions and $k_i = |k_0| \cos \theta_i (i = 1, 2)$ are the wavevector components parallel to the mean field, $B_0 e_z$, in which $\theta_1$ and $\theta_2$ are the angles between $k_0$ and $B_0 e_z$. We also assume $\delta B_{1x} = \delta B_{1y} = \delta B_1$, $\delta B_{2x} = \delta B_{2y} = \delta B_2$ and $\delta B_1 = \delta B_2 = \delta B$.

5.1. Results from numerical simulations

Our aim is to study the different particle behavior varying both the wave’s amplitudes $\delta B/B_0$ and the orientations $\theta_1$ and $\theta_2$. Particles are randomly injected in the simulation box with an initial velocity $v = 4v_A$ and $\beta = 10$.

Figs. 4 and 5 show the $x$- and $y$-components of particle velocity ($v_x$, top left, and $v_y$, top right) and $x$ and $y$ components of magnetic field seen by the particle (bottom left, and $B_y$, bottom right), for two different wave amplitude values, $\delta B/B_0 = 0.001$ and $\delta B/B_0 = 0.1$, respectively, using three different wave orientations:
Particle \( v_x \) (top left) and \( v_y \) (top right) velocities and \( B_y \) (bottom left) and \( B_x \) (bottom right) component of magnetic field seen by the particle. Particle initial total velocity and wave amplitude are \( v = 4v_A \) and \( \delta B/B_0 = 0.001 \). Different panels correspond to different wave orientations: 4(a) for \( \theta_1 = 0^\circ \) and \( \theta_2 = 0^\circ \), 4(b) for \( \theta_1 = 0^\circ \) and \( \theta_2 = 30^\circ \), 4(c) for \( \theta_1 = 0^\circ \) and \( \theta_2 = 180^\circ \).

Instead, considering two antiparallel waves, Fig. 4(c), the \( y \)-component of particle velocity is still in phase with \( -B_x \), but this is not true anymore for \( v_x \) and \( B_y \). This behavior is easily understood from the expressions of the fields:

\[
B_x = 2\delta B \cos(k_0 z), \quad B_y = -2\delta B \sin(k_0 z) \quad (19)
\]

for two parallel waves,

\[
B_x = 2\delta B \cos(k_0 z), \quad B_y = -2\delta B \cos(k_0 z) \quad (20)
\]

for two antiparallel waves.

While \( B_y \) is the same for both cases, in the antiparallel case \( B_x \) is proportional to \( \cos(k_0 z) \) and is not moving in phase with \( v_x \). As a consequence the resonance is broken, as evident in Fig. 4(c).
When the second wave is not moving parallel or antiparallel with respect to the first one ($\theta_2 = 30^\circ$, Fig. 6(b)), after some gyroperiods the magnetic field amplitude is modulated by the presence of the $k_1 = k_2$ term in the cosine and sine arguments of Eq. (18). Increasing the $\delta B/B_0$ value, we are moving far from the quasilinear approximation and the unperturbed solution of the Eq. (6) starts to be not a good approximation of particle motion in the presence of the perturbation. In Fig. 5, the same as Fig. 4 but for $\delta B/B_0 = 0.1$, the effect of a stronger perturbation on particle velocity is visible.

To gain some insight into the nature of gyroresonant interaction, we show particles’ motion in $v_i - v_\perp$ space, varying both the initial value of the cosine of pitch angle $\mu$ and wave amplitudes $\delta B/B_0$ for different wave orientations. Following the resonance condition, Eq. (15), different resonances occur for different values of particle parallel velocity, i.e., different $\mu$, when all particles are loaded with the same initial velocity $v_i$. The $l = 0$ resonance corresponds to $v_i = 0$; instead, other resonances are obtained for different $v_i$ depending on the $k_1$ value. These velocities are listed in Table 2 for the resonance $l = \pm 1$ for different angles. In Fig. 6, particles’ orbits in $v_i - v_\perp$ space are shown for $\delta B/B_0 = 0.1$ (Fig. 6(a)) and for $\delta B/B_0 = 0.1$ and $\delta B/B_0 = 0.2$ (Fig. 6(b)) using four different values of wave orientations:
- $\theta_1 = 0^\circ$ and $\theta_2 = 0^\circ$ (top left),
- $\theta_1 = 0^\circ$ and $\theta_2 = 30^\circ$ (top right),
- $\theta_1 = 0^\circ$ and $\theta_2 = 180^\circ$ (bottom left),
- $\theta_1 = 30^\circ$ and $\theta_2 = 210^\circ$ (bottom right).

Particles are injected with a different cosine of pitch angle $\mu$ close to the resonant value $\mu_{res} = 1/4$. Because the amplitude of the wave is large, adjacent resonances overlap and particles with an initial cosine of pitch angle different from $\mu_{res} = 1/4$ can also cover a big portion of the $v_i - v_\perp$ space, although the biggest spread is associated with $\mu = 1/4$ (blue line).

During the interaction with two parallel waves (Fig. 6(a) (top left)), the particles’ motions are centered around $v_i = 1v_\parallel$, the point which corresponds to the $l = -1$ resonance for both waves. If the particles are moving in the field of two obliquely propagating waves, they can cover a larger portion of $v_i - v_\perp$ space depending on the waves’ relative orientation. This behavior is evident in the other three plots of Fig. 6:
- $\theta_1 = 0^\circ$ and $\theta_2 = 30^\circ$ (top right): particles start to cover bigger distances in $v_i - v_\perp$. Moreover the motion is localized just in the positive side of $v_i - v_\perp$ space, because the resonance’s width due to both waves is stronger on one side of the circle.
- $\theta_1 = 0^\circ$ and $\theta_2 = 180^\circ$ (bottom left): the second wave is moving in the opposite direction with respect to the first one. The second resonance allows particles to go completely around the circle. The motion now is centered at $v_i \sim 0$, because two opposite resonances and two waves with the same amplitude are considered.
- $\theta_1 = 30^\circ$ and $\theta_2 = 210^\circ$ (bottom right): this situation is similar to the previous one, expect for the fact that in this case the motion is centered at $v_i \sim 0.57v_\parallel$, because resonances occur at $v_i = \mp 1.154v_\parallel$.

When waves with different amplitudes are considered, depending on which is the largest, the center of particles motion can move to the left or to the right of the circle, as evident in the Fig. 6(b). In this case the second wave has a double amplitude with respect to the first one, so the motion moves on the portion of the circle where resonances associated with the second wave are localized.

### Table 2

<table>
<thead>
<tr>
<th>$\theta$ (°)</th>
<th>$k_1 = k_0 \cos \theta$</th>
<th>$v_i = \mp (\beta/k_0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$k_0$</td>
<td>±1</td>
</tr>
<tr>
<td>30</td>
<td>$\pm k_0/2$</td>
<td>±1.154</td>
</tr>
<tr>
<td>180</td>
<td>$-k_0$</td>
<td>±1</td>
</tr>
<tr>
<td>210</td>
<td>$\pm k_0/2$</td>
<td>±1.154</td>
</tr>
</tbody>
</table>

6. The slab model

A particularly simple model of plasma turbulence is the so-called slab model [6,10]. Turbulence is assumed to be a sum of right and left hand circularly polarized, parallel propagating, nondispersive plane Alfvén waves. In this model the wave vector is parallel to the direction of the mean field and the fluctuations of the magnetic field are perpendicular to both the parallel wave vector and the mean field, as shown in Fig. 7(a).

From the definition of the slab field, the fluctuation depends only on the $z$-coordinate as clear in Fig. 7(b), where two different field lines in a pure slab turbulence are shown. Therefore, if we consider the slab fluctuation in the $x$-$y$ plane at each $z$ position, $b_\text{lab}$ is the same on that plane but different from the field on other planes as shown in Fig. 7(a).

For this study it is assumed that the turbulence is static in time for all practical purposes. This yields a turbulence spectrum that has finite width in $k$ but zero width in $\alpha$. 

![Graph showing particle motion](image-url)
6.1. Slab magnetostatic turbulent field generation

The test particle simulations are carried out in a unidimensional box of length \( L = 10000\Omega \). The magnetic field \( \mathbf{B}(z) \) is stored on a grid of spacing \( \Delta z = L/N_z \), where \( N_z \) is an even integer which we fixed at \( N_z = 2^{28} = 268,435,456 \). The magnetic field configuration is generated through a spectrum \( P(k) \) in \( k \)-space.

The grids in real space are then produced via inverse fast Fourier transform (FFT). For our one-dimensional field configuration, the turbulent magnetic field satisfies \( \delta \mathbf{B}_x(z) \hat{\mathbf{e}}_x + \delta \mathbf{B}_y(z) \hat{\mathbf{e}}_y \), with the full magnetic field given by:

\[
\mathbf{B}(z) = B_0 \hat{\mathbf{e}}_z + \delta \mathbf{B}(z),
\]

and \( \nabla \cdot \mathbf{B}(z) = 0 \), identically.

The vectors denoting the FFT of \( \delta \mathbf{B}(z_m) \) \( (m = 1, 2, 3, \ldots, N_z) \) are generated through

\[
\delta B_x(k_n) = [P(k_n)]^{1/2} e^{i\Phi_x},
\]

\[
\delta B_y(k_n) = [P(k_n)]^{1/2} e^{i\Phi_y},
\]

where \( k_n = 2\pi n/L \) is the discrete wavevector with spacing \( \Delta k = 2\pi/L; \Phi_x \) and \( \Phi_y \) are randomly phases; \( P(k) \) is the spectral shape function given by:

\[
P(k_n) = \begin{cases} 
p_{\text{slab}}(k_n) = C_{\text{slab}}[1 + (k_n l_z)^2]^{-5/6}, & \text{for } k_n < k_{\text{diss}} \\
p_{\text{diss}}(k_n) = C_{\text{diss}} \left( \frac{k_n}{k_{\text{diss}}} \right)^{-7/3}, & \text{for } k_n \geq k_{\text{diss}} \end{cases}
\]

where \( C_{\text{slab}} = 2\lambda_\text{c}^2 (\delta B_0^2)/B_0 \) is the normalization constant for the slab model; \( k_{\text{diss}} \) is the wavenumber at the beginning of the dissipation range; \( C_{\text{diss}} \) is the constant for the part of the spectrum corresponding to the dissipation range, determined by the fact that the spectrum must not have discontinuities in \( k \) space, i.e., setting \( p_{\text{slab}}(k_{\text{diss}}) = p_{\text{diss}}(k_{\text{diss}}) \),

\[
C_{\text{diss}} = C_{\text{slab}}[1 + (k_{\text{diss}} l_z)^2]^{-5/6}.
\]

The vectors of Fourier coefficients are zero-padded from \( N_{\text{max}} + 1 \) to \( N_z \), providing an extra level of smoothness to the fields by an effective trigonometric interpolation. In the simulations we set \( N_{\text{max}} = 6.7 \times 10^7 \). This high level of smoothness in the fields allows us the use of simple linear interpolation to evaluate the fields at the test particle position. The actual \( \delta B_x(z_m) \) and \( \delta B_y(z_m) \) are generated from the above discrete Fourier transforms through use of the inverse one-dimensional FFT.

6.1.1. Importance of scale separation

The spectrum generated by our numerical simulation is shown in Fig. 8. From the figure it is possible to distinguish several scales of importance characterized by different wavenumbers, labeled in Fig. 8 as \( k_{\text{min}}, k_{l_1}, k_{\text{diss}}, k_{\text{max}}, k_N \). As we showed in the previous section, the discrete wavenumber are given by \( k_n = 2\pi n/L \), where \( L = 10000 l_z \) is the boxlength, \( l_z = 1 \) is the coherence length for the slab spectrum, used as the characteristic length of the system, \( n = N_k \) are the number of points for the considered \( k_n \):

\[
k_n = \frac{2\pi n}{L} = (6.28 \times 10^{-4}) n \quad \text{and}
\]

\[
n = N_k = \frac{L}{2\pi} k = 1600k
\]

We summarize the values for \( k \) and \( N_k \) used in our simulations in Table 3 where:

- \( k_{\text{min}} = 2\pi /L \) is the minimum wave vector of the spectrum, corresponding with \( N_k = N_{k_{\text{min}}} = 1 \).
- \( k_{l_1} = 1/l_z = 1 \) is the threshold wave vector of the spectrum, corresponding with \( l_z \), that marks the beginning of the inertial range. Three decades of the energy containing scale, from \( k_{\text{min}} \) to \( k_{l_1} \) ensure turbulence homogeneity. \( l_z \), or \( \lambda_\text{c} = 0.747 l_z \) [4], is also the typical scale for pitch angle diffusion. Three decades of the inertial range, characterized by \( P(k) \propto k^{-5/3} \), well represent the solar wind case.
They are integrated for 1000 gyroperiods. Other parameters are the number of processors for the problem of 100 test particles moving in a slab realization with the spectrum as shown in Fig. 8. The particles are initialized in the simulation box with random position and an initial velocity $v = 10v_0$ with random gyrophase. They are integrated for 1000 gyroperiods. Other parameters are fixed, i.e., $\delta B/B_0 = 0.001$ and $\beta = 10^4$. The simulations are done on a machine with 48 cores and 251 GB of total memory.

In Fig. 9 the efficiency $E(P)$ (first plot) and the speed-up $S(P)$ (second plot) versus the number of CPUs $P$ are shown. The efficiency and the speed-up are calculated as $E(P) = T_1/P T(P)$ and $S(P) = PE(P)$, respectively, where $T_1$ is the execution time of 1 CPU and $T(P)$ is the execution time of $P$ CPU. The speed-up $S$ represents the factor by which the execution time is reduced on $P$ CPU. In the figure the red lines represent the data, the blue lines are used to show the ideal case, i.e., $S(P) \propto P$ and $T(P) \propto 1/P$.

### 7. Velocity space diffusion in slab turbulence model

Charged particles interacting with weak plasma turbulence diffuse in velocity space, i.e., the variance $\langle \Delta v_i^2(t) \rangle = \langle (v_i(t) - \bar{v}_i(t))^2 \rangle$ increases linearly with time (in the case of normal diffusion) for times exceeding the field’s autocorrelation time, $\tau_{ef}$. The angle brackets, $\langle \cdots \rangle$, denote the ensemble average, $v_i$ is the component of particle velocity parallel to the mean magnetic field direction and $\bar{v}_i(t) = \langle v_i(t) \rangle$. The autocorrelation time, defined as $\tau_{ef} = 1/(\omega - k_i \cdot B_{\parallel})$ [11], is the time for any initially smooth distribution of wave phases to relax to a uniform phase distribution through dispersion. In numerical experiments one typically uses the period during which the variance exhibits linear behavior with time to define the parallel diffusion coefficient, $D_{\parallel}$, as $\langle (\Delta v_i(t))^2 \rangle = 2D_{\parallel} t$, for some range of $t$ [12,4].

### 8. Test particle simulations and numerical results

Particles are randomly loaded in space throughout the simulation box with their initial velocities given by

$$v_x = v \sin \theta \cos \phi, \quad v_y = v \sin \theta \sin \phi, \quad v_z = v \cos \theta,$$

where $v$ is the magnitude velocity, $\theta$ is the initial pitch angle and $\phi$ is the gyrophase. Particles are loaded with a cold ring beam distribution in velocity space, wherein $v = 10v_0$ is held constant, $\sin \theta$ is set equal to $1 - \mu_0^2)^{1/2}$, where $\mu_0 = 0.6$ is the initial cosine of pitch angle identical for all the particles, and the initial gyrophase $\phi$ is randomly selected for each particle.

In order to show the validity of our numerical model, we give an example of a well-resolved simulation for particle pitch angle diffusion in random slab-like magnetic fields. Typical parameters used in the simulation are $(B_0^2)^{1/2}/B_0 = 0.01$ and $\beta = 50$. Fig. 10 shows the half variance of the cosine of pitch angle $\langle (\Delta \mu)^2 \rangle/2$ for this simulation. This figure demonstrates very good agreement between our simulations and the quasilinear prediction over the full duration of the simulations, about $8\tau_{ef}$. Indeed $\langle (\Delta \mu^2) \rangle$ evolves linearly with time, according to $\langle (\Delta \mu)^2 \rangle/2 = D_{\mu} \tau$ plotted in the same figure with a dot-dashed line.

### 8.0.3. Effects related to box size

One of the most important parameter in the simulation is the box size $L$. It must be chosen so that

- the simulated turbulent wave spectrum adequately approximates a continuum;
In order to illustrate some effects associated with a short box length, we perform various simulations starting with the same initial conditions but different box lengths: \( L = (50, 150, 500, 1000)\lambda \). Other parameters are \( \langle \delta B^2 \rangle^{1/2}/B_0 = 0.001, \beta = 500 \) and \( v = 10v_\parallel \). Fig. 11 shows the time evolution of the half pitch angle variance \( \langle (\Delta \mu)^2 \rangle/2 \) for the four different cases analyzed: \( L = 50\lambda \) is shown with \((-\cdots-\) line, \( L = 150\lambda \) with \((-\cdots)\) line; \( L = 500\lambda \) with \((-\cdots)\) line and \( L = 1000\lambda \) with \((-\cdots\cdots)\) line. The numerical results are compared with the quasi-linear prediction, \( \langle (\Delta \mu)^2 \rangle/2 \simeq D_{\mu\mu}t \) plotted in the same figure with the solid line. The best agreement with the theoretical prediction is obtained for the simulation with largest box length (\( L = 1000\lambda \), \((-\cdots)\) line), i.e., the highest \( k \)-space density of Fourier modes and consequently the smoothest distribution of wave phases. For all the other cases the agreement is relatively poor.

We believe that the main source of error for the shorter box lengths comes from the wider spacing of their Fourier components in \( k \)-space. Once the particles begin to spread in \( v_\parallel \) and, hence, decorrelate with their initial resonant wavenumber, they need to encounter other resonant \( k \)-modes, so that their motion becomes stochastic and leads to diffusion in velocity. In the continuum case a given particle will always find another wavenumber with which to resonate, provided there is finite power in a neighborhood of its original resonant wavenumber. This is expected to be typical of very large homogenous systems with essentially continuum wavenumber distributions of wave energy. However, in the finite case, when the density of discrete Fourier modes decreases, the likelihood that a particle finds a matching \( k \) and increases the importance of nonlinear effects, such as resonance broadening, decreases. Thus the discreteness of the Fourier spectrum of the simulated turbulence is plainly exposed and manifests itself in very poor agreement with the simulation results of the quasi-linear theory. This disagreement can manifest itself as either a subdiffusive (\( L = 50\lambda \)) or superdiffusive (\( L = 150\lambda, 500\lambda \)) trend relative to quasi-linear theory. Furthermore, since the case for \( L = 500 \) does not agree with theory and, on average, the distance traveled by a particle down the simulation box in the parallel direction is about \( 60\lambda \ll 500\lambda \), this rules out periodicity effects associated with the fields as the possible cause.

Fig. 12 illustrates pitch angle variance \( \langle (\Delta \mu)^2 \rangle \) (top panel) and particle mean square displacement \( \langle (\Delta y)^2 \rangle \) (bottom panel) in an extreme case, \( L = 10\lambda \) and \( v_\parallel = 6v_\lambda \), so that particles make more than 5 transits through the simulation box during the total duration of the simulation \( T = 10\tau_\lambda \). The numerical curve for \( \langle (\Delta \mu)^2 \rangle \) exhibits oscillations with a frequency equal to the box-crossing frequency. Each successive box crossing is depicted by a peak in the curve of \( \langle (\Delta y)^2 \rangle \). At the beginning these peaks are coherent because particle velocity in the parallel direction is strongly peaked around the initial value \( v_\parallel = 6 \) as the particles start to diffuse in pitch angle, the distribution in \( v_\parallel \) broadens and the trajectories begin to decorrelate, yielding a lack of coherence evident in the spatial variance after \( t \approx 5.5\tau_\lambda \). After \( t = 4\tau_\lambda \) \( \langle (\Delta \mu)^2 \rangle \) deviates from the theoretical curve. This regime could be termed diffusive too, but the diffusion coefficient differs
from that one predicted by quasilinear theory. At the same time
the spatial coherence is lost and the spatial oscillations are less
pronounced.

Another subtle, numerically important effect related with the
box size, is that the index of the Fourier component/coefficient
corresponding to the resonant wavenumber depends on the
normalized box length $L/\lambda$:

$$|k_{\text{res}}| = \frac{\Omega}{|v_i|} = \frac{\beta}{\mu(v/v_0)\lambda} \quad \text{(24)}$$

$$n_{\text{res}} = \frac{1}{2\pi} \frac{\alpha}{\mu(v/v_0)} L \quad \text{(25)}$$

where $n_{\text{res}}$ is the index of the resonant Fourier coefficient. Varying
the box length has the property varying $n_{\text{res}}$ proportionally and
varying the wavenumber spacing $\Delta k = 2\pi/L$ in inverse
proportions. Indeed a shorter box length is advantageous from
the point of view that it lowers the ratio $n_{\text{res}}/n_{\text{Nyq}}$, where $n_{\text{Nyq}}$ is the
$n$ corresponding to the Nyquist wavenumber. Consequently it
ensures plenty of wave power above the resonant wavenumber and
the sinusoidal Fourier mode corresponding to the resonant
wavenumber is well resolved spatially, reducing interpolation
errors. The high spatial resolution of the resonant mode gives rise
to good energy conservation and, consequently, high fidelity in
the particle orbit dynamics. Thus, the "Hamiltonicity" of the problem
is very nearly conserved.

Indeed, although the theory of pitch angle diffusion in
magnetostatic fields rigorously predicts no diffusion in particle
energy, i.e., particles are confined to a particular $H$-surface for all
time, finite diffusion in energy is difficult to avoid in the discrete
model. We illustrate the good energy conservation for low $L/\lambda$
in Fig. 13. Energy is conserved better when $L/\lambda = 25$ case
(lower curve) with respect to the case with $L/\lambda = 1000$ (upper
curve). Thus a compromise must be sought whereby the resonant
wavenumber is placed in a region of $k$-space where (i) the Fourier
mode corresponding to the resonant wavenumber is sufficiently
well resolved spatially so that the drift across $H$-surfaces is
minimized, and (ii) the interwavenumber spacing should be small
enough so that the resonance condition is easily satisfied, allowing
particles to diffuse stochastically along their constant $H$-surface.

8.0.4. Grid spatial resolution: interpolation errors

In order to illustrate the effect of field grid resolution, we
performed two simulations using a different number of grid points
$N$. Both simulations are designed so that the fields have identical
spectral content, i.e., the number of non-zero Fourier components

in both runs is $n = 2^{16} = 65536$. In the first run we set $N = n$.
Indeed, in the second run the Fourier coefficient array is zero-
padded up to $N = 2^{22} = 4194304$, which means the vector
of Fourier coefficients is of length $N = 2^{22}$ but there is nonzero
power only in the first $n = 2^{16}$ coefficients. The resonant mode
number does not depend on grid size $N$, and it is set equal to
$n_{\text{res}} \simeq 13263$ and $k_{\text{res}} = 2\pi n_{\text{res}}/L$ for both simulations. Thus the
only difference between these two runs is the spatial resolution or
smoothness of the fields. The run with the zero padded coefficient
array has a higher degree of "trigonometric interpolation" and
much smoother fields.

Results for $\langle (\Delta \mu)^2 \rangle/2$ are shown in Fig. 14 with a solid line: the upper solid line represents the case $N = N_2 = 2^{24}$, the lower one is for $N = N_1 = 2^{16}$. The quasilinear prediction in the $k$-space between adjacent Fourier modes $\Delta k = 2\pi/L$ is the same
for each run. This rules out any effects related to the density of
Fourier modes in $k$-space. However, by zero padding the Fourier
array to a larger $N$ value, we obtain better resolved fields. In effect
we are restricting (or bandwidth-limiting) the largest wavenumber
in the system to be $k_s = 2\pi n_s/L$, but the effective sampling rate
is increased by increasing the Nyquist wavenumber $k_{\text{Nyq}} = \pi N/L =
\pi/\Delta z$, where $N/2 \gg n_s$. Thus the smallest spatial variations of
the fields occur on scale lengths of the order of $l_{\text{min}} \sim L/n_s$, but
the fields are resolved down to scale lengths of the order of $\Delta z =
L/N = (n_s/N)l_{\text{min}}$.

In other words, we are using the inverse of the sampling theorem [1]: if a function $h(z)$, sampled in space at an interval $\Delta z$, is known to be bandwidth limited to wavenumbers smaller in magnitude than $k_s$ and if it turns out that the Fourier coefficients for $k > k_s$ are all identically zero, then the function is completely
determined by its discrete samples $h_n(n \Delta z)$. Indeed in this case we
have [1]

$$h(z) = \Delta z \sum_{n=-\infty}^{\infty} h_n \sin[k_{\text{Nyq}}(z - n \Delta z)] \frac{\sin(\pi z)}{\pi (z - n \Delta z)}.$$ 

At first sight the idea of a bandwidth limited signal might seem
artificial, but it has a valid physical reason. Indeed at least within
the context of MHD turbulence, frequencies and wavenumbers are
bandwidth limited by the basic assumptions governing the validity
of MHD. These are $\omega < \Omega_i$ and $k \rho_i < 1$, where $\Omega_i$ is the ion
gyrofrequency and $\rho_i = v_{thi}/\Omega_i$ is the ion thermal gyroradius. A
left hand circularly polarized mode, whose electric field vector has
the same sense of gyration about the mean magnetic field as an
ion, has a resonance at $\Omega_i$, so it is bandwidth limited physically through ion cyclotron damping at $\omega(k_i) = k_i v_A \simeq \Omega_i$ (related to the maximum parallel wavenumber).

The smoother fields, as a result of these extra zero Fourier coefficients, allow us to use safely simple linear interpolation to evaluate the electromagnetic fields at the particle positions. The advantage of the approach from the point of view of CPU cycles is the speed. The disadvantage is, of course, the extra computer memory required.

9. Conclusions

We have presented a new version of the Streamline test-particle code. It has the ability to solve generally the first order ODE, given by Eq. (1), where the quantity $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ is an $N$-dimensional generalized position that could in fact represent both static field-lines or particle trajectories. For this reason the Streamline code can be used to study multiple and different physical problem, in contrast with other simple test-particle codes. The re-designed Streamline structure allows to compute not just particle and/or field-lines positions and velocities at each steps of the simulations, but also the instantaneous fields values as seen by the particles. This is helpful, for example, to compute the instantaneous value of particle magnetic moment [13], but other applications are possible too.

Several standard cases are built in present, choosing of ODE’s (field line or charged particle orbit equations) and the magnetic field input model (slab, slab plus 2D, data read from file). Multiple test cases are also installed in the code. Indeed Streamline Version 4 is capable of handling analytic fields, tabulated fields from external files, or no fields at all, so long as Eq. (1) is fully defined. These fields are expected to be electromagnetic, but this is not assumed.

This code is a MPI parallel implementation and is able to maintain a local relative accuracy of $r_{acc} = 10^{-9}$ at each simulation step. Different test particles moving in a constant magnetic field $B = B_0 e_z$ (see Figs. 1(a) and 2); particles moving in a constant magnetic field $\mathbf{B} = B_0 e_z$ plus a constant electric field $\mathbf{E} = E_0 e_x$ (Fig. 1(b)); particles moving in a circularly polarized wave field (Fig. 3) are shown in this paper, confirming that maintaining this accuracy our results maintain a relative error of order $10^{-9}$ at the end of the simulations. When particle trajectories are integrated, the test simulations are stopped after $10^9$ gyroperiods, a reasonably longer time to study realistic physical problems.

After the accuracy test, we study the resonance behavior of a single particle in the field of two circularly polarized electromagnetic waves. Particle velocity components and the magnetic fields seen by the particle (see Figs. 4 and 5) are computed in order to confirm the general view of the resonant interaction. In addition to gaining more insight into the nature of gyroresonant interaction varying the waves amplitude and their relative orientation, particle motion in $v_1 - v_2$ space is analyzed in details (see Fig. 6).

Finally we investigate the velocity diffusion phenomenon when a distribution of particles interacts with a broadband turbulent slab spectrum (see Eq. (22) and Fig. 8). Apart from the obvious limitation that the spectrum is purely one dimensional, it is constructed to correspond roughly to features of solar wind spectra observed by single spacecraft, where the fully three dimensional spectrum is in effect reduced to a one dimensional form. We consider three decades of energy containing scale, three decades of the inertial range and another two decades of the dissipation range. After that there are almost another two decades of zero-padding, which are important for the trigonometric interpolations and for the smoothness of the field. The resulting magnetic field is stored on a homogeneous grid of spacing $\Delta z = L/N_z$, where $N_z$ is an even integer fixed at $N_z = 2^{28} = 268,435,456$ points. Moreover, the Streamline code can be used in conjunction with an adaptive mesh refinement method. Indeed, thanks to the code structure, it is easy to add a subroutine to compute, for example, the current density, and it is possible to define a non-homogeneous grid thicker in the regions where the resulting current density is higher.

In the last section of the paper, we concentrate our study on the periodicity effects associated with the length size (see Figs. 11 and 12), that can lead to “fake” diffusion behavior. Other issues concerning energy conservation (Fig. 13) and interpolation errors (Fig. 14), related to the spatial resolution of the grid on which the discrete fields are interpolated are discussed in detail too. Thus, final suggestions are given on how to design good numerical simulations, whose purpose is the study of particle diffusion phenomena, as well as magnetic field line diffusion and so on.

References